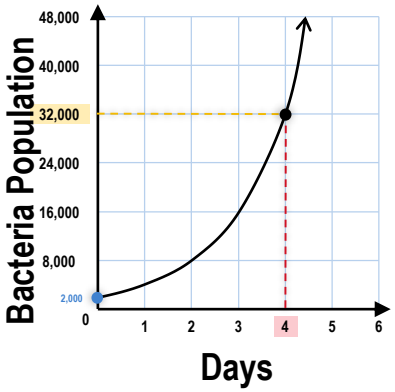
We will solve problems using exponential equations in one variable. | Concept Development

A **growth** describes a situation in which a quantity ***increases***.

- *In an equation, an **exponential growth** has a **constant factor** greater than 1.*

A population of bacteria **doubles** every day. If the population of bacteria starts at **2,000**, **how many days** would it take for it to reach **32,000**?



$$\frac{2{,}000(2)^x}{2{,}000} = \frac{32{,}000}{2{,}000}$$

$$(2)^x = 16$$

$$(2)^x = (2)^4$$

$$x = 4$$

It would take **4 days** for the population of bacteria to reach **32,000**.

A **growth** describes a situation in which a quantity ***increases***.

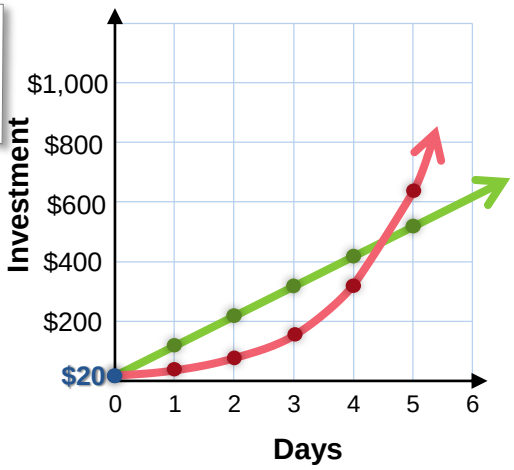
- *In an equation, an **exponential growth** has a **constant factor** greater than 1.*

Linear growth versus **Exponential growth Investing $20.**

An investment **increases $100** a day.

Every day, your investment **doubles** the amount of the previous day.

Day 1	$120	$40
Day 2	$220	$80
Day 3	$320	$160
Day 4	$420	$320
Day 5	$520	$640



CFU

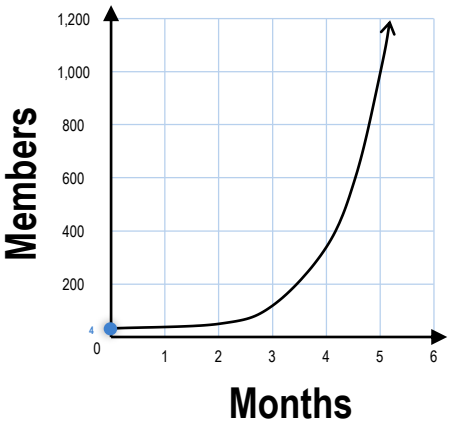
Which equation below represents an exponential growth? How do you know?

A $4000(3)^x = 108000$ B $22400\left(\frac{1}{4}\right)^x = 1400$

In your own words, how is exponential growth different from linear growth?

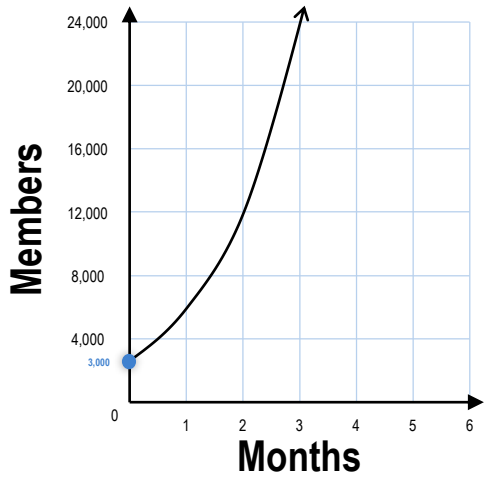
1. Write an exponential equation to represent the problem, if needed.
2. Divide each side of the equation by the **initial amount**.
3. Solve for the variable.
4. Interpret the solution.

1. Mia is creating a new social networking website with three of her classmates. She knows that if each of the members recruits 2 more members each month, the number of members will triple every month. How long will it take for her to reach 972 members?



$4(3)^x = 972$

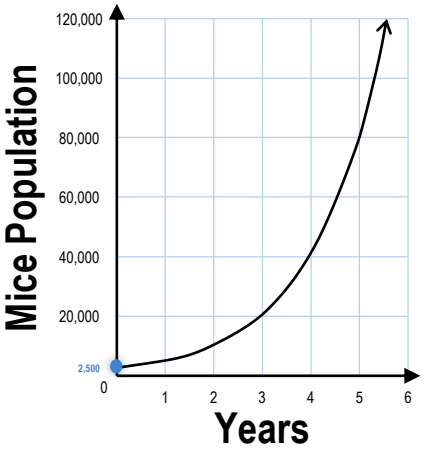
2. Mia's social networking website has reached 3,000 members. She estimates the membership will now begin to double every month. How long will it take for the website to increase to 24,000 members?



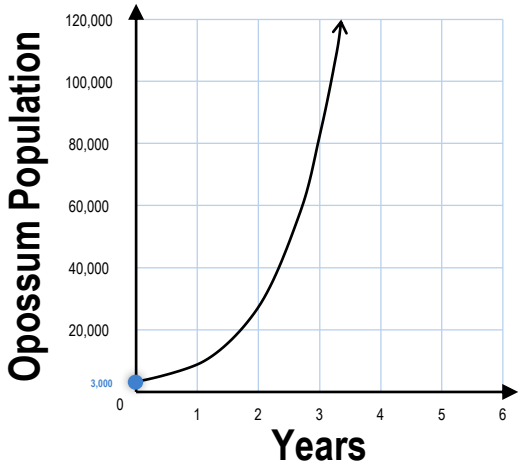
$3{,}000(2)^x = 24{,}000$

1. Write an exponential equation to represent the problem, if needed.
2. Divide each side of the equation by the **initial amount**.
3. Solve for the variable.
4. Interpret the solution.

3. In a given environment, scientists estimate that removing snakes in the area would allow the population of mice to double every year. If there are currently 2,500 mice, how long would it take for the population of mice to reach 80,000 if snakes were removed?



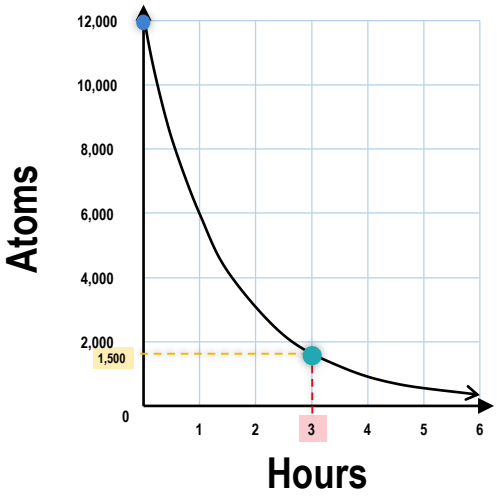
4. In a given region, it is hypothesized that extinction of a population of owls would result in annual$_2$ tripling of the opossum population. If there are currently 3,000 opossums, how long would it take for the opossum population to reach 81,000 if the owls became extinct?



A **decay** describes a situation in which a quantity ***decreases***.

- *In an equation, an **exponential decay** has a **constant factor** less than 1.*

The **half-life**$_3$ of a radioactive substance is one hour. If the initial amount of the substance is **12,000 atoms**, **how many hours** would it take for the substance to decay to **1,500 atoms**?



$$\frac{12,000\left(\frac{1}{2}\right)^{x}}{12,000} = \frac{1,500}{12,000}$$

$$\left(\frac{1}{2}\right)^{x} = \frac{1}{8}$$

$$\left(\frac{1}{2}\right)^{x} = \left(\frac{1}{2}\right)^{3}$$

$$x = 3$$

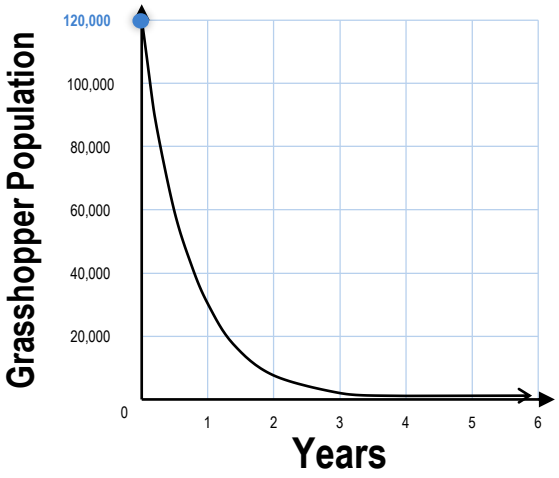
It would take **3 hours** for the substance to decay to **1,500 atoms**.

CFU

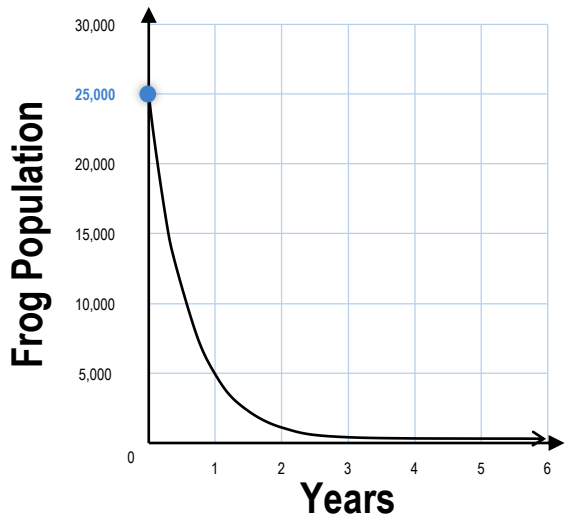
What is the difference between exponential growth and an exponential decay?

1. Write an exponential equation to represent the problem, if needed.
2. Divide each side of the equation by the **initial amount**.
3. Solve for the variable.
4. Interpret the solution.

5. After the rapid growth in the rat population, it is predicted that every year the grasshopper population will be reduced to one-fourth. How long will it take for the grasshopper population to drop to 1,875 in the region, given that it is now 120,000?

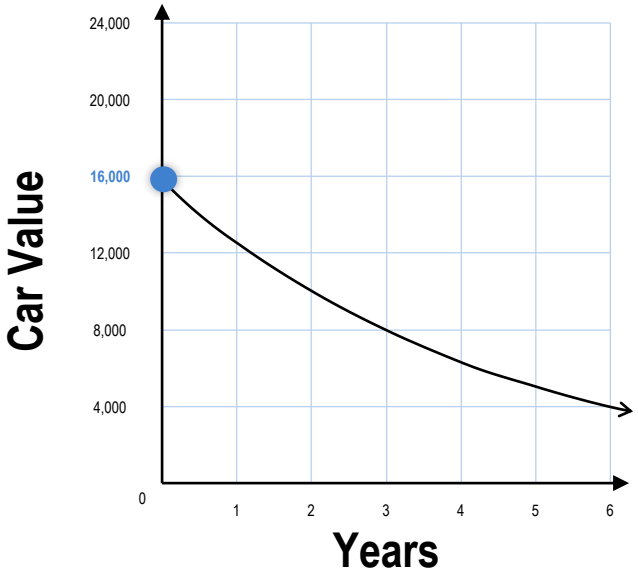


6. A rapid growth in the opossum population would cause competition over insects with the frog population. As a result, it is expected that the frog population would be reduced to one-fifth every year. How long would it take for the frog population in the region to drop to 200, given that it is now 25,000?

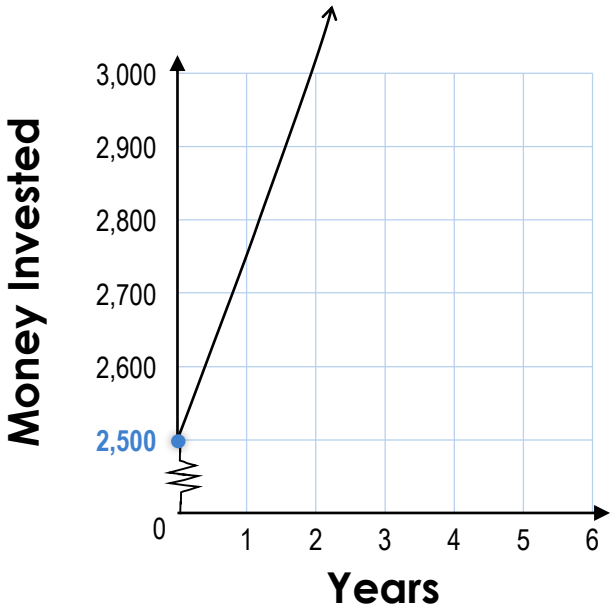


1. Write an exponential equation to represent the problem, if needed.
2. Divide each side of the equation by the **initial amount**.
3. Solve for the variable.
4. Interpret the solution.

7. This week, Thomas bought a new car for $16,000. Every year, the car depreciates$_4$ by 20%. How long will it take for the car to be worth $8,192?



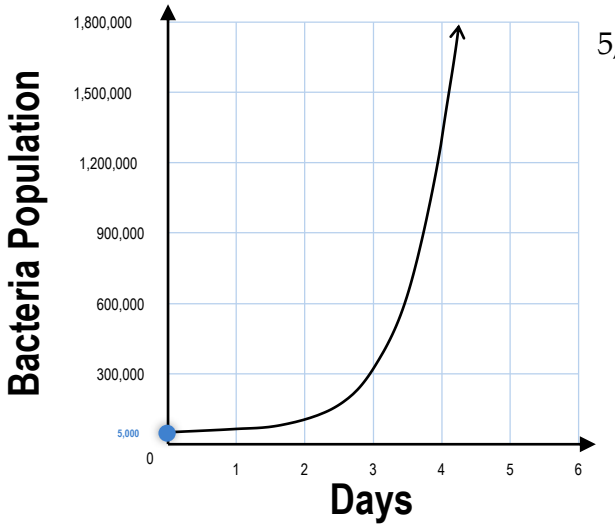
8. For graduation, Karen receives $2,500 as a gift from her grandfather. She invests all the money, earning annual interest at a rate of 10%. How long will it take for Karen to have $3,025 invested?



We will solve problems using exponential equations in one variable. | **Closure**

1. Write an exponential equation to represent the problem, if needed.
2. Divide each side of the equation by the **initial amount**.
3. Solve for the variable.
4. Interpret the solution.

1. A population of bacteria quadruples every day. If the population of bacteria starts at 5,000, how many days will it take for it to reach 1,280,000?



$5{,}000(4)^x = 1{,}280{,}000$

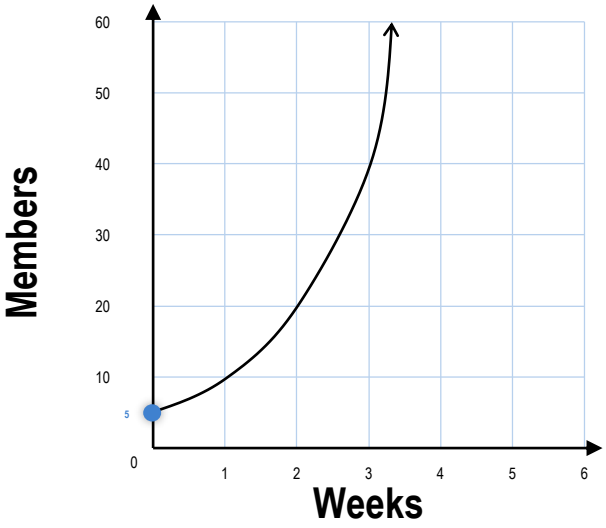
Writing

Write a word problem that describes exponential growth. Use a scenario about investment of money.

What did you learn today about solving problems using exponential equations in one variable?

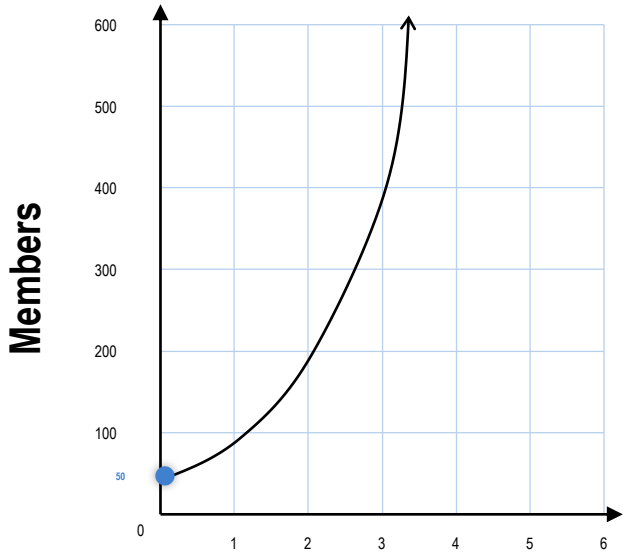
Write the problem as an exponential equation, divide each side by the initial amount, then solve for the variable.

1. Nancy is creating a new club at school with four of her friends. She knows that if each of the members recruits 1 more member each week, the number of members will double every week. How long will it take for her club to reach 40 members?



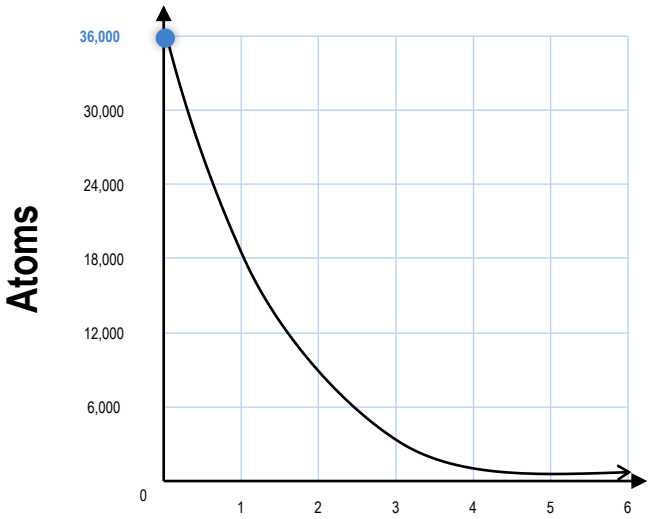
$5(2)^x = 40$

2. Nancy's new club has reached 50 members. She estimates the membership will now begin to double every semester. How long will it take for the club to reach 200 members?

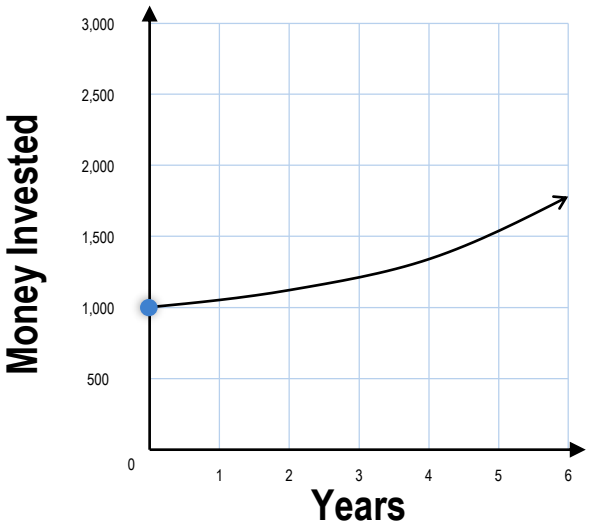


Write the problem as an exponential equation, divide each side by the initial amount, then solve for the variable.

3. The half-life of a radioactive substance is one hour. If the initial amount of the substance is 36,000 atoms, how many hours would it take for the substance to decay to 1,125 atoms?

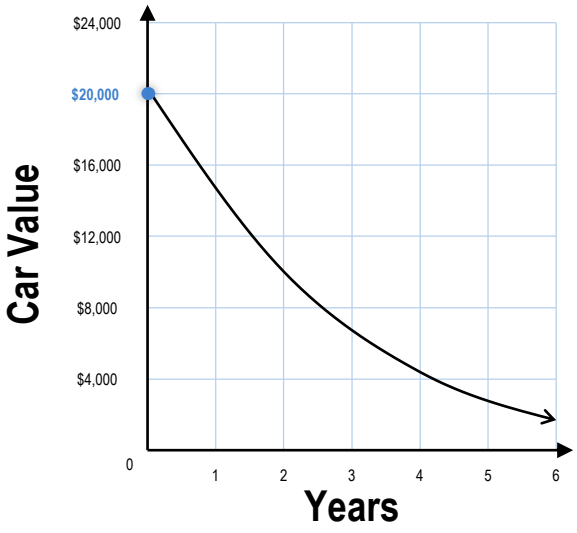


4. Holly earned $3,000 working a summer job. She invested $1,000 of her earnings at a 10% annual interest rate. If Holly does not withdraw any of the money, how long will it take for her to have $1,331 invested?



Write the problem as an exponential equation, divide each side by the initial amount, then solve for the variable.

2. This week, Ronald bought a new car for $20,000. Every year, the car depreciates by 30%. How long will it take for the car to be worth $4,802?



Reading

Read the problems. Choose Yes or Not to indicate whether each statement is true about the problem.

1. A population of bacteria quadruples every day. If the population of bacteria starts at 10,000, how many days will it take for it to reach 640,000?

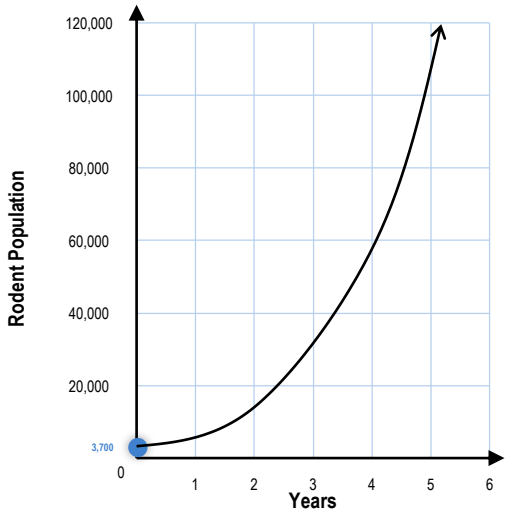
A The problem is an **exponential growth**.	O Yes O No
B The **initial amount** of bacteria is **10,000**.	O Yes O No
C The equation that represents the problem is **$10{,}000(4)^x = 640{,}000$**.	O Yes O No
D It will take **4 days** for the bacteria to reach **640,000**.	O Yes O No

2. The half-life of a radioactive substance is one hour. If the initial amount of the substance is 100,000 atoms, how many hours would it take for the substance to drop to 6,250 atoms?

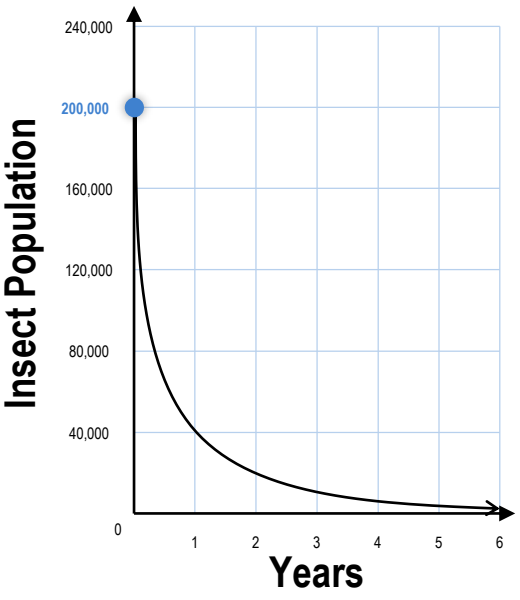
A The problem is an **exponential decay**.	O Yes O No
B The **final amount** of atoms is **100,000**.	O Yes O No
C The equation that represents the problem is **$100{,}000(2)^x = 6{,}250$**.	O Yes O No
D It will take **4 hours** for substance to drop to **6,250 atoms**.	O Yes O No

Write the problem as an exponential equation, divide each side by the initial amount, then solve for the variable.

1. In a given environment, scientists estimate that removing owls in the area would allow the population of rodents to double every year. If there are currently 3,700 rodents, how long would it take for the population of rodents to reach 118,400 if owls were removed?



2. After the rapid growth in the rodent population, it is predicted that every year the insect population will be reduced to one-fifth as a direct result of the increase of rodents. How long would it take for the insect population to drop to 1,600 in the region, given that it is now 200,000?



Writing

Write word problems.

For a situation that could be modeled with an **exponential growth**.

For a situation that could be modeled with an **exponential decay**.

Write the problem as an exponential equation, divide each side by the initial amount, then solve for the variable.

1. The half-life of a radioactive substance is two hours. If the initial amount of the substance is 50,000 atoms, how many hours would it take for the substance to decay to 12,500 atoms?